

Bollin Primary School

Progression in written methods for calculations

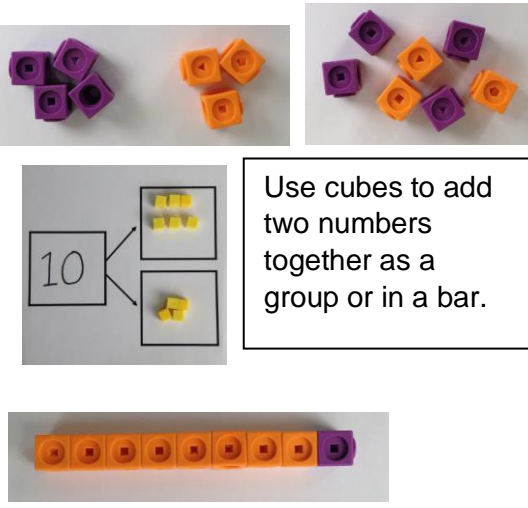
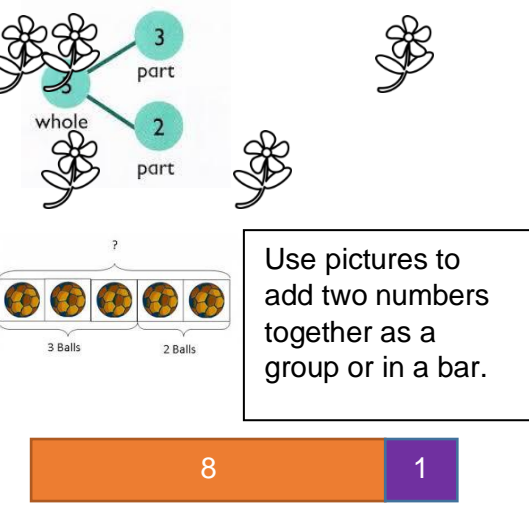
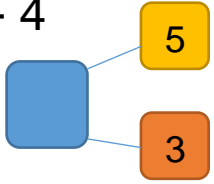

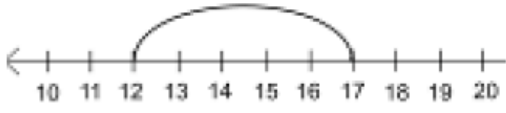

- For written calculations it is essential that there is a progression which culminates in one method.
- The individual steps within the progression are important in scaffolding children's understanding and should not be rushed through.
- Practical equipment, models and images are crucial in supporting children's understanding.

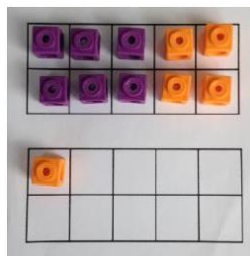
Bollin Primary School



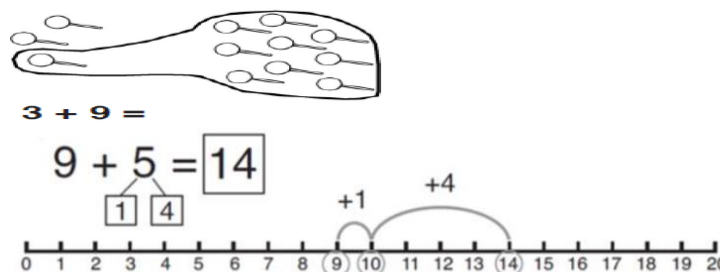
Growing hearts and minds together

Addition

| Objective and Strategies | Concrete | Pictorial | Abstract |
|--|---|---|---|
| <p>Combining two parts to make a whole: part-whole model</p> |  <p>Use cubes to add two numbers together as a group or in a bar.</p> |  <p>Use pictures to add two numbers together as a group or in a bar.</p> | <p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> |
| <p>Starting at the bigger number and counting on</p> |  <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p> | <p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p> | <p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p> |
| <p>Regrouping to make 10.</p> |  <p>$6 + 5 = 11$</p> | <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p> | <p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p> |



Start with the bigger number and use the smaller number to make 10.

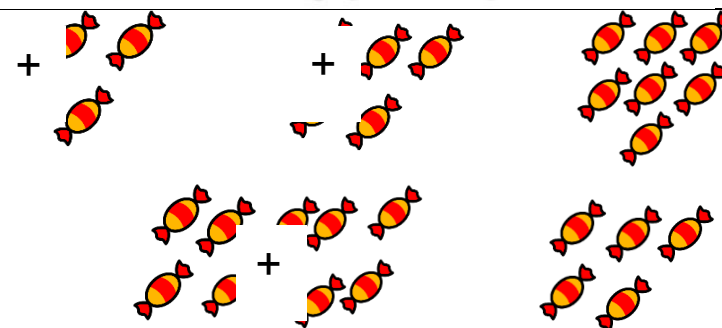


Adding three single digits

$4 + 7 + 6 = 17$
Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



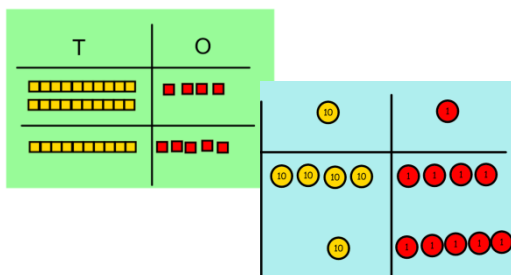
Add together three groups of objects. Draw a picture to recombine the groups to make 10.

$$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ \quad \quad \quad 10 \\ = 17 \end{array}$$

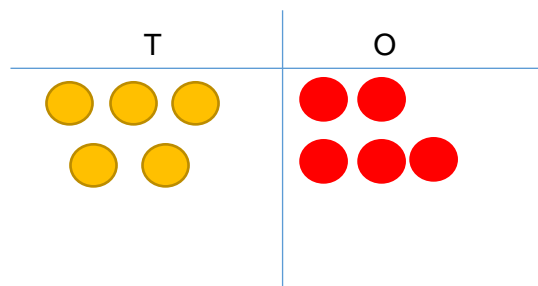
Combine the two numbers that make 10 and then add on the remainder.

Column method- no regrouping

$24 + 15 =$
Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



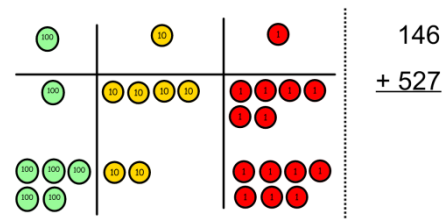
Calculations

$$21 + 42 =$$

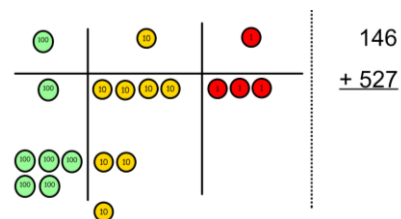
$$\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$$

Column method-regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

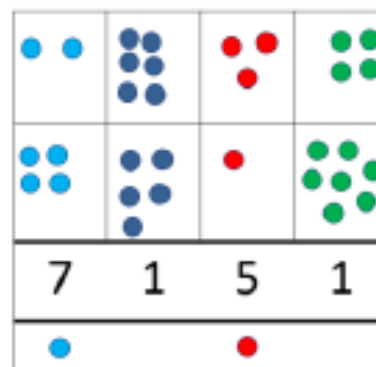


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

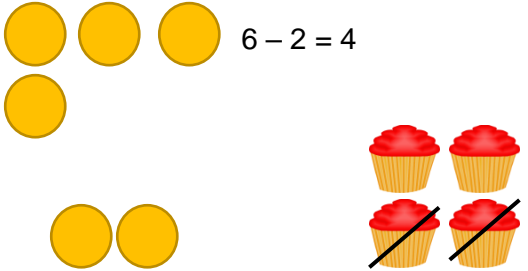
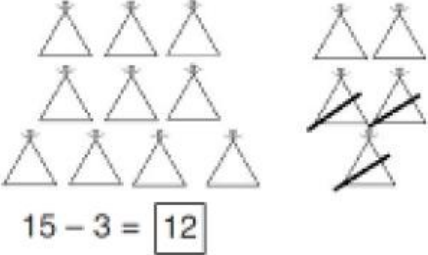


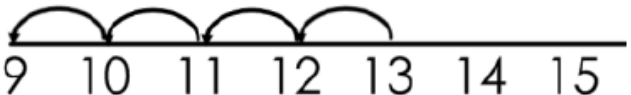
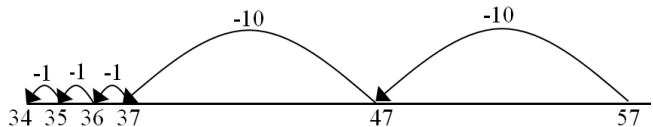
$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Subtraction

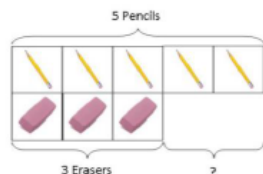
| Objective and Strategies | Concrete | Pictorial | Abstract |
|--------------------------------|---|---|---|
| <p>Taking away ones</p> | <p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p> | <p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p> | <p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p> |
| <p>Counting back</p> | <p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p>  | <p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p> | <p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p> |

Find the difference

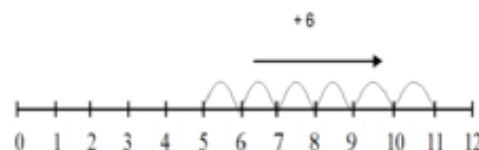
Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference



Use basic bar models with items to find the difference

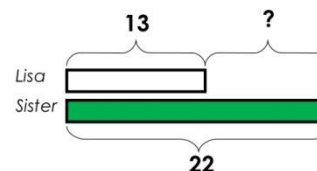


Count on to find the difference.

Comparison Bar Models

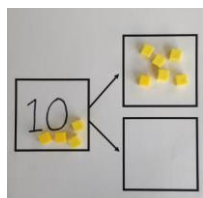
Draw bars to find the difference between 2 numbers.

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Part Part Whole Model

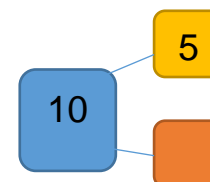
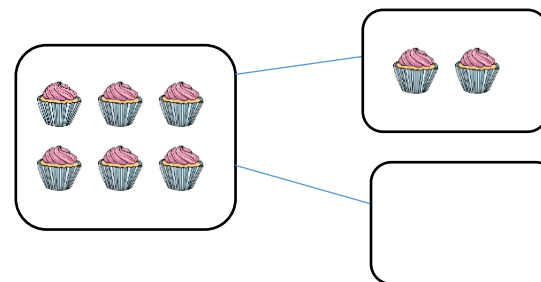


Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

$$10 - 6 =$$

Use a pictorial representation of objects to show the part part whole model.



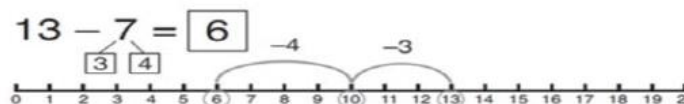
Move to using numbers within the part whole model.

Make 10

$$14 - 9 =$$



Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.



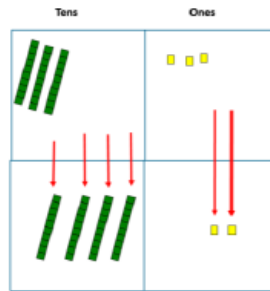
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

$$16 - 8 =$$

How many do we take off to reach the next 10?

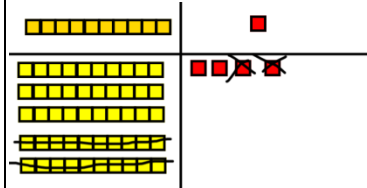
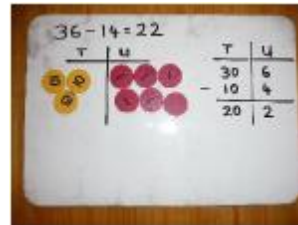
How many do we have left to take off?

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$\begin{array}{r} 47 - 24 = 23 \\ \begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array} \end{array}$$

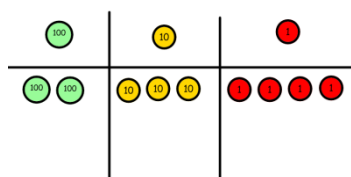
This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

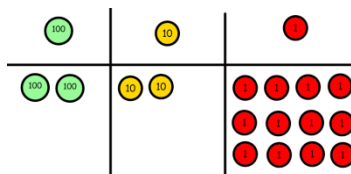
Make the larger number with the place value counters



Calculations

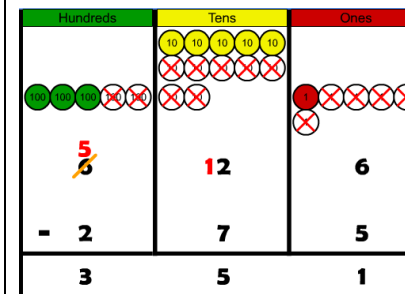
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

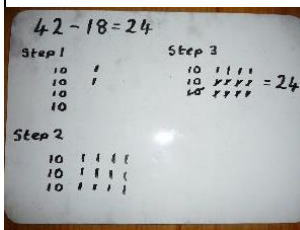
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

$$\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{r} 800 \quad 30 \quad 6 \\ - 200 \quad 50 \quad 4 \\ \hline 500 \quad 80 \quad 2 \end{array} \end{array}$$

Children can start their formal written method by partitioning the number into clear place value columns.



When confident, children can find their own way to record the exchange/regrouping.

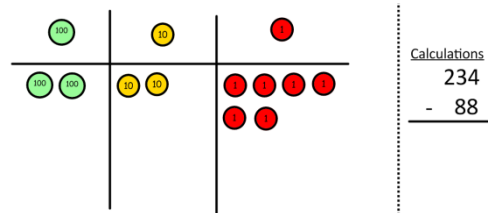
Just writing the numbers as shown here shows that the child understands the method and

knows when to exchange/regroup.

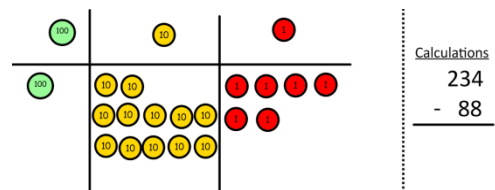
$$\begin{array}{r} 728 - 582 = 146 \\ \begin{array}{r} 700 \quad 20 \quad 8 \\ - 500 \quad 80 \quad 2 \\ \hline 100 \quad 40 \quad 6 \end{array} \end{array}$$

Moving forward the children use a more compact method.

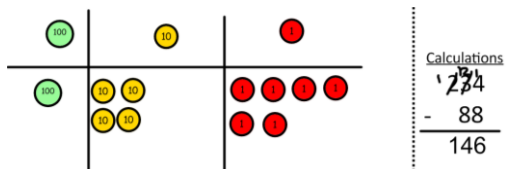
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

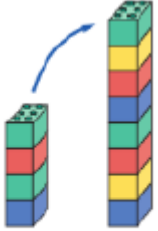

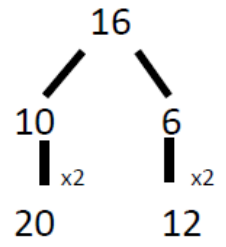
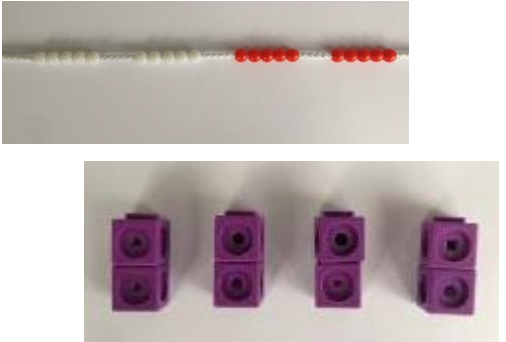
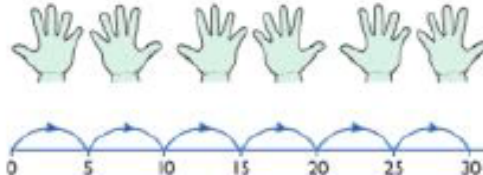


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.



This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad 0 \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$


Multiplication

| Objective and Strategies | Concrete | Pictorial | Abstract |
|------------------------------|--|--|--|
| Doubling | <p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p> | <p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p>  |  <p>Partition a number and then double each part before recombining it back together.</p> |
| Counting in multiples |  <p>Count in multiples supported by concrete objects in equal groups.</p> |  <p>Use a number line or pictures to continue support in counting in multiples.</p> | <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> |

Repeated addition





3 + 3 + 3

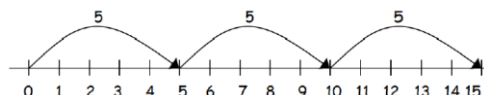


Use different objects to add equal groups.

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



5 + 5 + 5 = 15

Write addition sentences to describe objects and pictures.



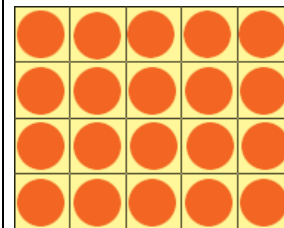

2 + 2 + 2 + 2 + 2 = 10

Arrays- showing commutative multiplication

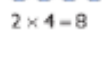
Create arrays using counters/ cubes to show multiplication sentences.




Draw arrays in different rotations to find **commutative** multiplication sentences.


4 × 2 = 8



2 × 4 = 8



2 × 4 = 8



4 × 2 = 8

Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

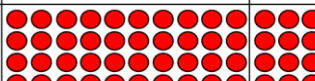

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$



Grid Method

Show the link with arrays to first introduce the grid method.

| | | |
|---|---|---|
| x | 10 | 3 |
| 4 |  |  |




4 rows
of 10
4 rows
of 3

Move on to using Base 10 to move towards a more compact method.

| | | |
|---|---|---|
| x | T | U |
| |  |  |
















4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

| | | |
|---|---|---|
|  |  |  |
| | | |
| | | |
| | | |
| | | |
















Calculations
 4×126

Fill each row with 126.

| | | |
|---|---|---|
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Calculations
 4×126

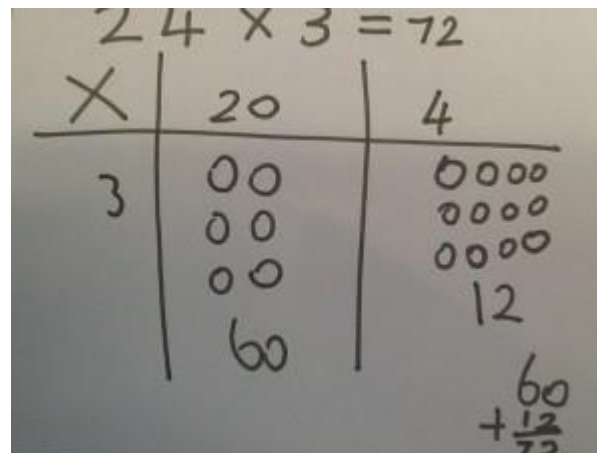
Add up each column, starting with the ones making any exchanges needed.

| | | |
|---|---|---|
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Then you have
your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

| | | |
|---|-----|----|
| x | 30 | 5 |
| 7 | 210 | 35 |

$$210 + 35 = 245$$

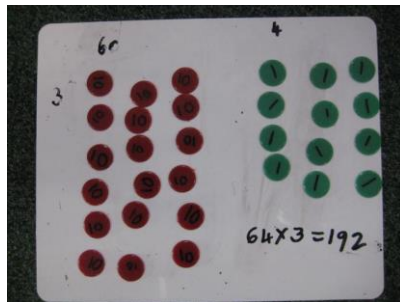
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

| | | |
|----|-----|----|
| | 10 | 8 |
| 10 | 100 | 80 |
| 3 | 30 | 24 |

| | | | | |
|----|-------|------|-----|----|
| x | 1000 | 300 | 40 | 2 |
| 10 | 10000 | 3000 | 400 | 20 |
| 8 | 8000 | 2400 | 320 | 16 |

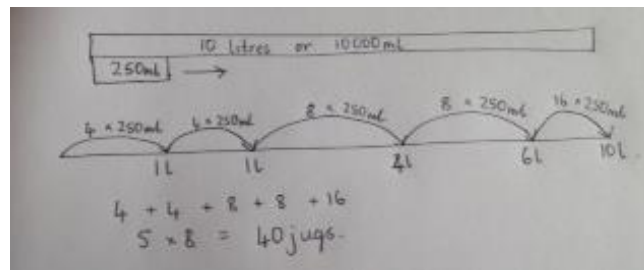
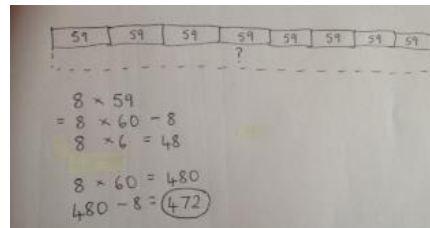
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.


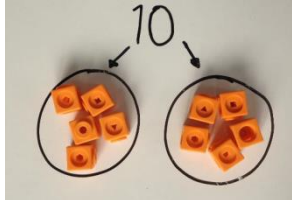

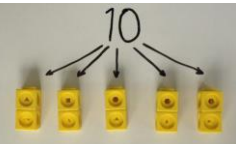

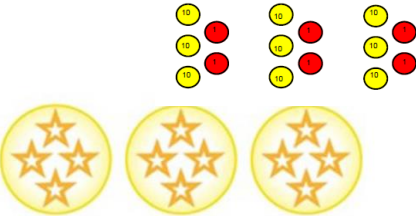
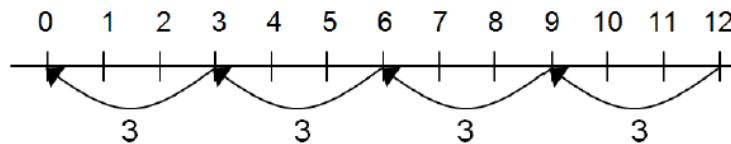
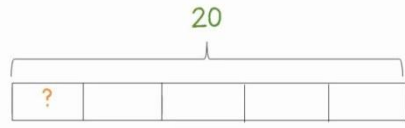
$$\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$$

$$\begin{array}{r} 74 \\ \times 63 \\ \hline 12 \\ 210 \\ 240 \\ + 4200 \\ \hline 4662 \end{array}$$

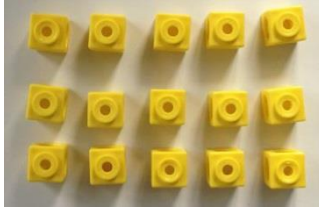
This moves to the more compact method.

$$\begin{array}{r} 231 \\ \times 18 \\ \hline 13420 \\ 10736 \\ \hline 24156 \\ 1 \end{array}$$

Division

| Objective and Strategies | Concrete | Pictorial | Abstract |
|------------------------------------|---|--|---|
| <p>Sharing objects into groups</p> |  <p>I have 10 cubes, can you share them equally in 2 groups?</p>  | <p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $8 \div 2 = 4$ </div> | <p>Share 9 buns between three people.</p> $9 \div 3 = 3$ |
| <p>Division as grouping</p> | <p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>   $96 \div 3 = 32$  | <p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$ | $28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p> |

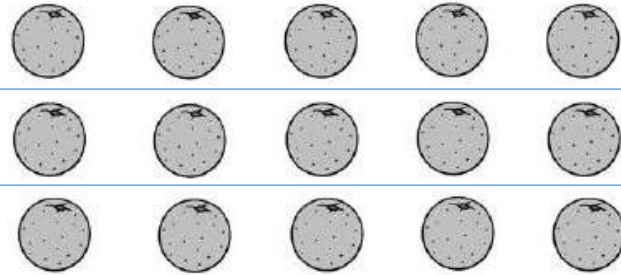
Division within arrays



Link division to multiplication by creating an array and thinking about the

number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



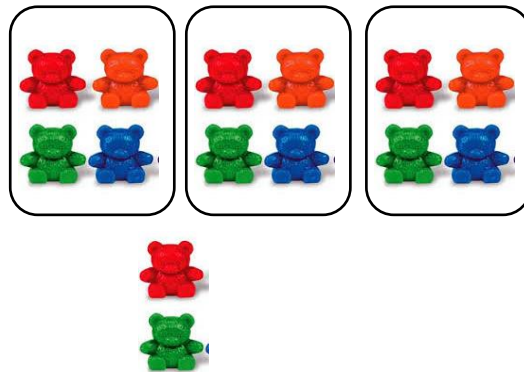
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

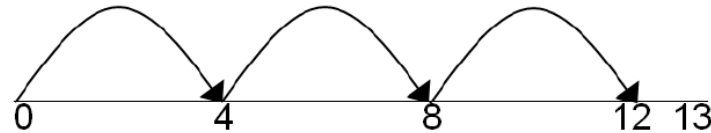
$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Division with a remainder

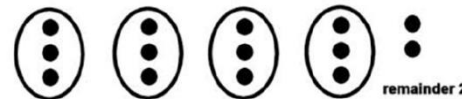
$14 \div 3 =$
 Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



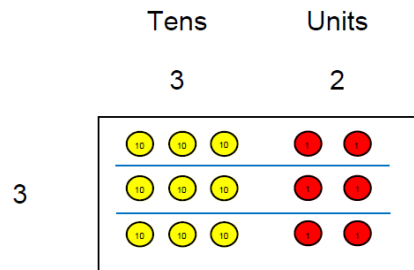
Draw dots and group them to divide an amount and clearly show a remainder.



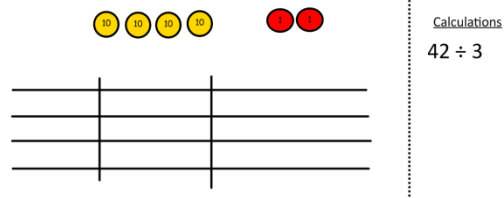
Complete written divisions and show the remainder using r.

$29 \div 8 = 3 \text{ REMAINDER } 5$
 ↑ ↑ ↑ ↑
 dividend divisor quotient remainder

Short division

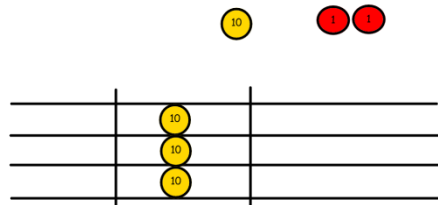


Use place value counters to divide using the bus stop method alongside

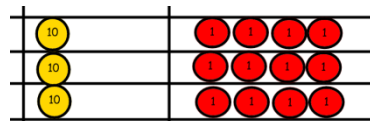


$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

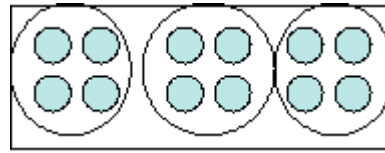


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 4872} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$